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EXPERIMENTAL STUDY OF SPECTRA CORRESPONDING TO FRICTION STRESS AND THE THIRD STATISTICAL MOMENTS IN FULLY DEVELOPED TURBULENT PIPE FLOW

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An analysis of experimental results on the behavior of spectra corresponding to the diagonal components of the Reynolds stress tensor [1] and a consideration of the similarity properties of a generalized model of the spectrum of an isotropic flow [2, 3] demonstrate the existence of a more or less universal representation for these spectral distributions. In the energy-significant interval the wavenumber is normalized to the longitudinal integral correlation scale. Nonuniformity and anisotropy of the flow introduce systematic deviations into the spectral distributions.

The use of the integral scale as a characteristic length of similarity is built into the von Karman equation [4] and in Rotta's derivation [4] of the transport equation for the integral correlation scale. An intriguing topic in connection with this problem is the behavior of the spectra corresponding to higher moments. Here we investigate the third moments measured by a hot-wire anemometer using an x-shaped probe.

The measurements were carried out for fully developed turbulent flow in a straight pipe of diameter 0.06 m at an axial velocity of 10 m/sec; the kinematic viscosity was $\nu = 1.4\cdot10^{-5}$ m^2/sec, and the Reynolds number determined from the average velocity was $\mathrm{Re} = 3.47\cdot10^4$. The friction velocity $v_f = 0.433$ m/sec was determined from curves plotted for fully developed pipe flow [6]. The apparatus and methodological considerations are discussed in [7-9]. A standard R61 sensor with two filaments of length $l = 1.2\cdot10^{-3}$ m and standard DISA hardware units were used; the signals were recorded on a digital tape recorder and were processed on a Plurimat-S computer. Special processing programs were developed to encompass the entire dynamic spectral range. The characteristics of the linearizers were selected with a view toward creating identical linear dependences for both filaments. The velocity–voltage conversion nonlinearity did not exceed 1%. The sensitivity was chosen on the basis of the condition that a signal attaining four standard deviations should be transmitted without distortion.

The fast Fourier transform and a procedure described in [10] are used to estimate the spectrum. The number of realizations is $M = 1024$ (sometimes 256), each m-th realization has a duration T ($N = 2048$ readings in a realization). The discretization frequency is $f_1 = 5000$ Hz. A smoothing Hannah window is used. The one-sided cross-spectral densities are obtained by averaging over all realizations:

$$E_{xy}(f) = \Big\{ \sum_{m=1}^{M} {}_mG_{xy}(f) \Big\} / M.$$

In each realization we have

$${}_mG_{xy}(f) = 2\, X_m\overline{(f)}\, Y_m(f) / T,$$

where $\overline{X_m(f)}$ is the complex conjugate of the Fourier transform of the signal $x_m(t)$, and $Y_m(f)$ is the value of the Fourier transform of the signal $y_m(t)$. We assume that

$$x_m(t) = \begin{cases} (U_1)_m - \langle U_1 \rangle, \\ (U_2)_m - \langle U_2 \rangle, \end{cases}$$

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TABLE 1

r'	$\langle u_x^2 \rangle$	$\langle u_r^2 \rangle$	$\langle u_\varphi^2 \rangle$	$\langle u_x u_r \rangle$	$\langle U_x \rangle$	Re_λ	Λ_0	$\eta\ 10^4$
0	0,125	0,0865	0,083	0	10,0	65	0,01	1,75
0,2	0,171	0,0986	0,100	0,036	9,82	81	0,012	1,66
0,4	0.278	0,136	0,149	0,072	9,38	106	0,015	1,49
0,6	0,404	0 167	0,205	0,108	8.72	120	0,015	1,31
0 8	0.555	0 197	0.262	0.143	7.66	120	0,013	1,12

TABLE 2

r'	$-\langle u_x^3 \rangle / v_*^3$			$-\langle u_x u_r^2 \rangle / v_*^3$			$-\langle u_r u_x^2 \rangle / v_*^3$		
	this study	[12]	[13]	this study	[12]	[13]	this study	[12]	[13]
0	0,26	0,25	0,39	0,11	0,10	0,15	−0,01	0	−0,05
0,2	0,47	0 50	0,57	0,15	0,12	0 18	0,17	0,16	0,20
0,4	0,78	0,69	0,88	0,20	0.17	0 21	0,28	0,27	0,34
0,6	0 86	0,69	1,03	0 20	0,18	0,22	0,32	0,34	0,37
0 8	0,61	0,69	0.87	0,16	0,15	0,11	0,29	0,39	0,38

$$y_m(t) = \begin{cases} [(U_1)_m - \langle U_1 \rangle]^2 - \langle u_1^2 \rangle, \\ [(U_1)_m - \langle U_1 \rangle][(U_2)_m - \langle U_2 \rangle] - \langle u_1 u_2 \rangle, \\ [(U_2)_m - \langle U_2 \rangle]^2 - \langle u_2^2 \rangle, \end{cases}$$

where, for example, $\langle U_1 \rangle$ and $\langle u_1^2 \rangle = \frac{1}{NM} \sum_k U_{1k}^2 - \langle U_1 \rangle^2$ are the mean values over the points of all realizations.

We transform to wavenumbers k by the Taylor formula $k = 2\pi f / \langle U_1(r') \rangle$ (r' is the relative radial coordinate).

The size of the two-filament probe prevents us from obtaining the values of the spectrum in the region of wavenumbers $k' = k\eta \geq 0.2$ [$\eta = (\nu/\varepsilon)^{1/4}$ is the Kolmogorov microscale]. When the probe filament has the length corresponding to the wavenumber $k' = 0.1$, the error estimated by the procedure in [11] is 10%. Given the same value of k', the error due to discretization of the signal can attain 20%. The rms random error for the friction stress spectrum is 10% under the given experimental conditions and is clearly large for the spectra of third moments. Additional averaging is therefore carried out over frequencie⁻ in an interval of width 0.3 in the scale $\log(k)$.

The spectrـ of the third moments were measured for velocity fluctuations $u_x = u_1$ and $u_r = u_2$. The corresponding spectra are associated with indices $x - 1$, $r - 1$, and $\varphi - 3$; for example, the spectrum corresponding to friction stress is denoted by E_{12}. The first logical step is to attempt to represent the spectra in dimensionless form with the wavenumber normalized to the "isotropic" longitudinal integral correlation scale Λ_0 calculated from the local values of the fluctuation energy $\langle E \rangle = \langle u_i^2 \rangle / 2$ and the energy dissipation rate $\varepsilon = 15\varepsilon_{11}$ [2]:

$$\Lambda_0/\eta = Re_0^{1/2}[2.47 + 0.081 Re_0^{1/2}(Re_0^{1/2} - 1)].$$

Here $Re_0 = \lambda_0(2\langle E \rangle/3)^{1/2}/\nu$ is the turbulent Reynolds number, and λ_0 is the Taylor (transverse) microscale, $\lambda_0^2 = 10\nu\langle E \rangle/\varepsilon$.

Table 1 gives the flow data and the calculated quantities (velocity in m/sec and linear dimensions in m).

The one-point third moments divided by v_*^3 are given in Tables 2 and 3. The most complete investigation of the one-point third moments for the investigated flow is reported in [12, 13]. The third moments are most sensitive to the conditions under which fully developed flow is attained [12]. These conditions were strictly observed in the measurements. The deviations of several moments from zero on the axis indicates a measurement error of 0.01-0.05. The measured moments $\langle u_3 u_1^2 \rangle / v_*^3$ and $\langle u_3^3 \rangle / v_*^3$ vary within the same limits. We also compare the results of [12] for Re = $1.72 \cdot 10^4$ and [13] for Re = (9-25)$\cdot 10^4$. The entries in Table 2 are taken from graphs of the average values and then scaled. Of course, this operation in-

TABLE 3

r	$-\langle u_r^3\rangle/v_*^3$			$-\langle u_x u_\varphi^2\rangle/v_*^3$	
	this study	[12]	[13]	this study	[12]
0	−0,01	0	−0,03	0,113	0,106
0,2	0,097	0,105	0,107	0,16	0,15
0,4	0,16	0,19	0,18	0,23	0,20
0,6	0,16	0,21	0,17	0,24	0,19
0,8	0,12	0,22	0,13	0,13	0,11

TABLE 4

r	$-s_1$				$-s_2$		
	this study	[12]	[13]	[14]	this study	[12]	[13]
0	0,498	0,373	0,462	0,50/0,56	−0,008	0	−0,064
0,2	0,550	0,50	0,486	0,62/0,54	0,271	0,191	0,208
0,4	0,438	0,354	0,385	0,52/0,45	0,291	0,241	0,245
0,6	0,271	0,200	0,267	0,38/0,30	0,201	0,169	0,169
0,8	0,119	0,123	0,140	0,19/0,13	0,123	0,098	0,105

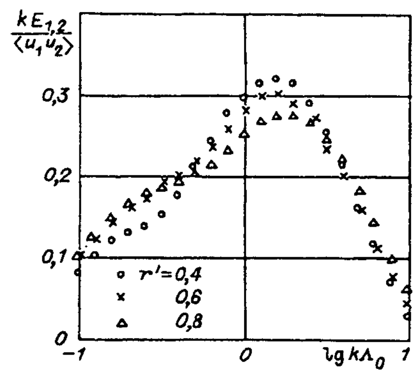


Fig. 1

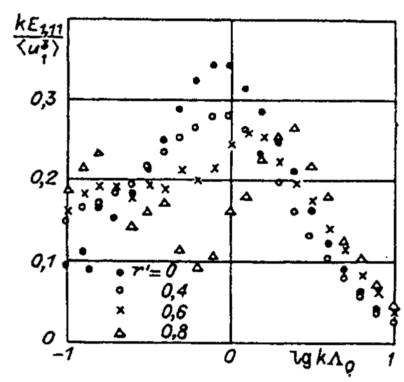


Fig. 2

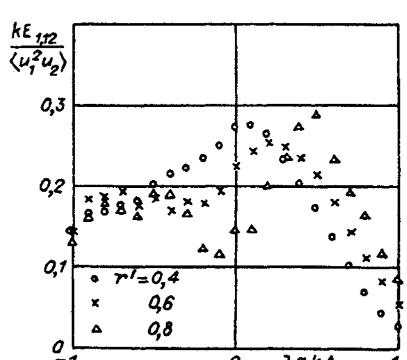
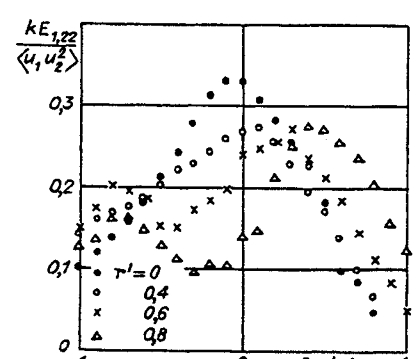


Fig. 3



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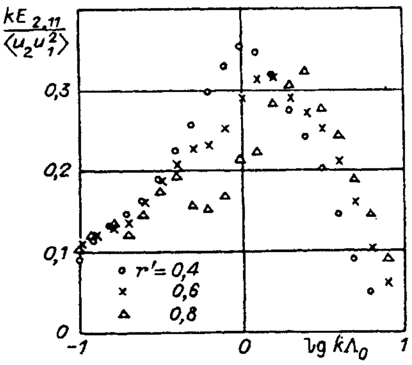

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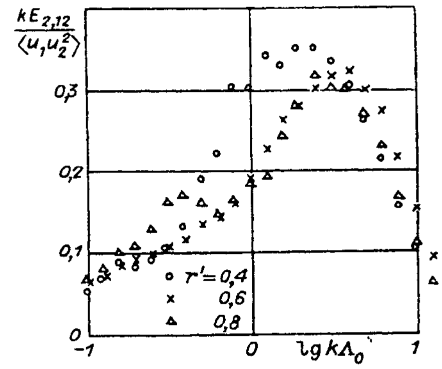

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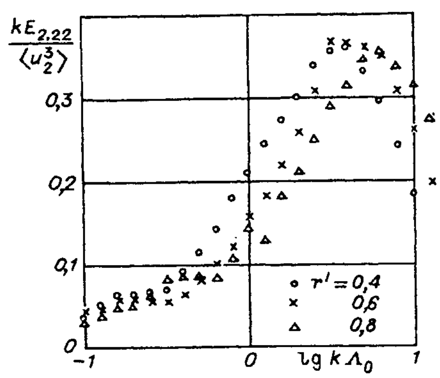

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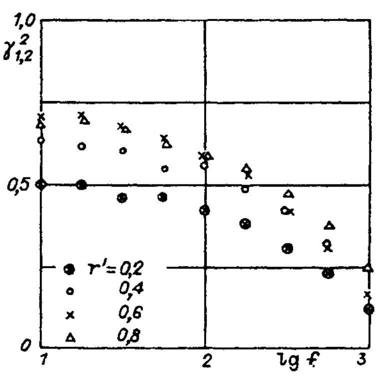

Fig. 8

troduces additional error. Data on the momental skewnesses $s_1 = \langle u_1^3 \rangle / \langle u_1^2 \rangle^{3/2}$ and $s_2 = \langle u_2^3 \rangle / \langle u_2^2 \rangle^{3/2}$ taken from Moscow Physicotechnical Institute (MFTI) studies and from [12-14] are compared in Table 4. In the case of [14] the entries in the denominator position correspond to a Reynolds number of $8 \cdot 10^4$, and those in the numerator position correspond to $4 \cdot 10^4$. The data of [15] exhibit considerable deviation from these values.

By and large, the deviations of the third moments according to the compared results are less than or equal to 25% up to a relative tube radius of 0.8. The scatter broadens closer to the tube wall. It is clearly important to bear in mind that influence of different values of the Reynolds number is smoothed out in the comparison. The error of determination of the friction velocity exerts a strong influence.

The objective of the comparison is to confirm that the experimental data on the one-point third moments in the present study give satisfactory agreement with and complement published data, justifying the data reliability for spectral measurements. Figure 1 shows the spectrum corresponding to the friction stress in dimensionless form. Figures 2-7 give the spectra of the third moments in dimensionless form. Figure 8 shows the dependence of the coherence factor

$$\gamma_{1,2}^2(f) = E_{12}^2(f) / [E_1(f) E_2(f)]$$

on the frequency f.

The spectral curves are clustered about certain universal distributions. The situation is similar to the case of the second moments. Turbulent pipe flow becomes strongly inhomogeneous and anisotropic in closer proximity to the wall. Systematic distortions appear in the spectral distributions in interaction with the inhomogeneous, anisotropic flow field. They can be taken into account by the further elaboration of turbulent transport theory. The advent of systematic distortions of the spectra for the friction stress and variances of the velocity fluctuations has been noted previously [13, 16-19].

On the whole, the data and concepts of an approximate universal expression for the spectra of the third moments should be useful in the formulation of a semiempirical theory of turbulent transport.

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